MAC 1114 - Trigonometry
Formula Sheet

Angles, Arc Length, and Circular Motion

1. DMS Conversions

1 Degree (1º): 1 minute (60')

1 minute (1'): 60 seconds (60'')

2. Deg-Rad conversion

Deg - RAD: deg × $\frac{\pi}{180}$

RAD - Deg: RAD × $\frac{180}{\pi}$

3. Circles

θ must be in RAD

Arc length (s): $s = r\theta$

Area of a sector (A): $A = \frac{1}{2}r^2\theta$

4. Circular Motion (Speed)

Linear speed (v): $v = \frac{d}{t}$ = r · ω
How fast a point moves along the arc

Angular speed (ω): $\omega = \frac{\theta}{t}$
How fast the central angle changes (θ in RAD)

Right Triangle Trigonometry

1. Trigonometric functions

sin θ = $\frac{\text{opposite}}{\text{hypotenuse}}$ csc θ = $\frac{\text{hypotenuse}}{\text{opposite}}$ csc θ = $\frac{1}{\sin\theta}$

cos θ = $\frac{\text{adjacent}}{\text{hypotenuse}}$ sec θ = $\frac{\text{hypotenuse}}{\text{adjacent}}$ sec θ = $\frac{1}{\cos\theta}$

tan θ = $\frac{\text{opposite}}{\text{adjacent}}$ cot θ = $\frac{\text{adjacent}}{\text{opposite}}$ cot θ = $\frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$ tan θ = $\frac{\sin\theta}{\cos\theta}$

Pythagorean Theorem: $h^2 = a^2 + b^2$

2. Complementary angles

sin θ = cos (90º - θ) csc θ = sec (90º - θ)

cos θ = sin (90º - θ) sec θ = csc (90º - θ)

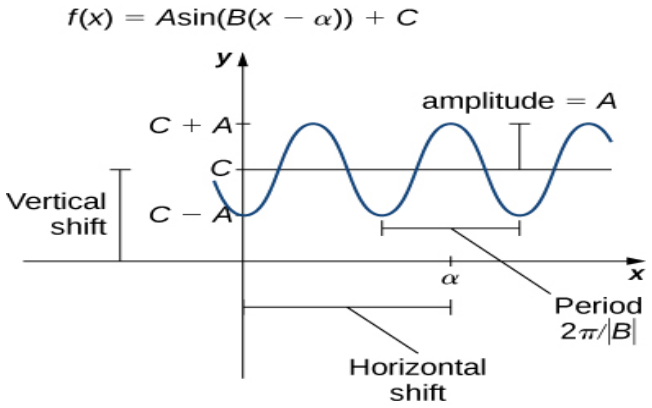
tan θ = cot (90º - θ) cot θ = tan (90º - θ)

Graphs

1. Equation of a trigonometric graph

$y = A \sin(\omega x - \varphi) + \beta$ A= amplitude φ = horizontal shift

$y = A \cos(\omega x - \varphi) + \beta$ ω= omega → Period (P) = $\frac{2\pi}{\omega}$ β = vertical shift



2. Asymptotes for reciprocal functions (and tan)

Tan (θ) → $\theta \neq \frac{\pi}{2}$ Cot (θ) → θ ≠ 0; π; 2π

Sec (θ) → $\theta \neq \frac{\pi}{2}$ Csc (θ) → θ ≠ 0; π; 2π

These are the reference or starting asymptotes. Because trig functions are periodic, the asymptotes will repeat every π radians from these angles listed above.

Inverse

1. Inverse Definition

For every trigonometric function
If θ = *sin (y)* then *y* = sin^{-1} (θ)

$\sin^{-1}(\sin(\theta)) = \theta$

2. Restriction Intervals

Sin / Csc
$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$ → $[-\frac{\pi}{2}, \frac{\pi}{2}]$ csc undefined at y=0

Cosine / Sec

$0 \leq y \leq \pi \rightarrow [0, \pi]$

sec undefined at y=$\frac{\pi}{2}$

Tan

$-\frac{\pi}{2} < y < \frac{\pi}{2} \rightarrow (-\frac{\pi}{2}, \frac{\pi}{2})$

Cot

$0 < y < \pi \rightarrow (0, \pi)$

[ ] include the endpoint
( ) exclude the endpoint

Trigonometric Identities

1. Pythagorean Identities

$\sin^2(x) + \cos^2(x) = 1$

$\tan^2(x) + 1 = \sec^2(x)$

$\cot^2(x) + 1 = \csc^2(x)$

2. Even – Odd identities

$\sin(-\theta) = -\sin(\theta)$ $\quad\quad$ $\csc(-\theta) = -\csc(\theta)$

$\cos(-\theta) = \cos(\theta)$ $\quad\quad$ $\sec(-\theta) = \sec(\theta)$

$\tan(-\theta) = -\tan(\theta)$ $\quad\quad$ $\cot(-\theta) = -\cot(\theta)$

3. Sum and difference formulas

$\cos(\alpha + \beta) = \cos(\alpha) \cdot \cos(\beta) - \sin(\alpha) \cdot \sin(\beta)$

$\cos(\alpha - \beta) = \cos(\alpha) \cdot \cos(\beta) + \sin(\alpha) \cdot \sin(\beta)$

$\sin(\alpha + \beta) = \sin(\alpha) \cdot \cos(\beta) + \cos(\alpha) \cdot \sin(\beta)$

$\sin(\alpha - \beta) = \sin(\alpha) \cdot \cos(\beta) - \cos(\alpha) \cdot \sin(\beta)$

$\cos\left(\frac{\pi}{2} - \theta\right) = \sin(\theta)$

$\sin\left(\frac{\pi}{2} - \theta\right) = \cos(\theta)$

$$\tan(\alpha + \beta) = \frac{\tan(\alpha) + \tan(\beta)}{1 - \tan(\alpha)\tan(\beta)}$$

$$\tan(\alpha - \beta) = \frac{\tan(\alpha) - \tan(\beta)}{1 + \tan(\alpha)\tan(\beta)}$$

4. Double Angle formulas

$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$ $\quad \sin^2(\theta) = \frac{1-\cos(2\theta)}{2}$ $\quad \tan(2\theta) = \frac{2\tan(\theta)}{1-\tan^2(\theta)}$

$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$

$\cos(2\theta) = 1 - 2\sin^2(\theta)$ $\quad \cos^2(\theta) = \frac{1+\cos(2\theta)}{2}$ $\quad \tan^2(\theta) = \frac{1-\cos(2\theta)}{1+\cos(2\theta)}$

$\cos(2\theta) = 2\cos^2(\theta) - 1$

5. Half Angle formulas

$\sin^2\left(\frac{\alpha}{2}\right) = \frac{1-\cos(\alpha)}{2}$

$\cos^2\left(\frac{\alpha}{2}\right) = \frac{1+\cos(\alpha)}{2}$

$\tan^2\left(\frac{\alpha}{2}\right) = \frac{1-\cos(\alpha)}{\sin(\alpha)} = \frac{\sin(\alpha)}{1+\cos(\alpha)}$

6. Product to Sum and Sum to Product

$\sin(\alpha)\sin(\beta) = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$

$\cos(\alpha)\cos(\beta) = \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]$

$\sin(\alpha)\cos(\beta) = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$

$\sin(\alpha) + \sin(\beta) = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$

$\sin(\alpha) - \sin(\beta) = 2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}$

$\cos(\alpha) + \cos(\beta) = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$

$\cos(\alpha) - \cos(\beta) = -2\sin\frac{\alpha-\beta}{2}\sin\frac{\alpha+\beta}{2}$

Applications of Trigonometric Functions

1.Law of Sine

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$

A + B + C = 180º

2. Law of Cosine

$c^2 = b^2 + a^2 - 2ab \cdot \cos(C)$

$b^2 = a^2 + c^2 - 2ac \cdot \cos(B)$

$a^2 = b^2 + c^2 - 2bc \cdot \cos(A)$

3. Simple Harmonic Motion

$d(t) = a\cos(\omega t)$ or $d(t) = a\sin(\omega t)$
Being *d(t)* the distance and *t* the time elapsed.

$f = \frac{1}{T} = \frac{\omega}{2\pi}$
Being *f* the frequency and *T* the period.

$d(t) = ae^{-\frac{bt}{2m}}\cos\left(\sqrt{\omega^2 - \frac{b^2}{4m^2}}t\right.$
For a damped motion. Damped → added friction → decreasing amplitude (*e*)

Polar Coordinates

(r,θ) → polar coordinates
(x,y) → rectangular coordinates

1.Conversion

$x = r\cos(\theta)$ $\quad$ $y = r\sin(\theta)$

$r^2 = x^2 + y^2$ $\quad$ $\tan(\theta) = \frac{y}{x}$ if $x \neq 0$

$r = y$ $\quad$ $\theta = \frac{y}{x}$ if $x = 0$

2. Polar equations

$a = r\sin(\theta)$
is a horizontal line that lies *a* units above/below the pole (depending on sign)

$a = r\cos(\theta)$
is a vertical line that lies a units to the right/left of the pole.

- Cardioids
  $\mathrm{r} = a(1 + \cos(\theta))$
  $\mathrm{r} = a(1 + \sin(\theta))$
- Limaçons
  $\mathrm{r} = a + b\cos(\theta)$
  $\mathrm{r} = a + b\sin(\theta)$
- Rose
  $r = a\cos(n\theta)$
  $r = a\sin(n\theta)$
- Lemniscates
  $r^2 = a^2\cos(2\theta)$
  $r^2 = a^2\sin(2\theta)$