MAC 1114 – TRIGONOMETRY
FINAL EXAM REVIEW

1. Sketch the angle $\theta = -210°$ in standard position. Then, find its reference angle and one positive coterminal angle.

2. A right triangle has a hypotenuse of length 17 and one leg of length 15. If θ is the angle adjacent to the leg of length 15, evaluate all six trigonometric functions for θ.

3. Convert the following angle measures:

   a) 315º to RAD
   b) $-\frac{2\pi}{3}$ to DEG

4. Identify the quadrant where θ lies if $\sin(\theta) < 0$ and $\cos(\theta) > 0$

5. Solve the following equation for all solutions in the interval $[0, 2\pi)$

$$2\sin(\theta) + \sqrt{3} = 0$$

6. Plot the point $(4, \frac{5\pi}{3})$ on a polar coordinate system. Then, convert this point into rectangular coordinates.

7. Solve the triangle where A=40º , B=60º , c=12. Round side lengths to two decimal places.

8. Find two angles — one positive and one negative — that are coterminal with 110°

9. A central angle of $\frac{\pi}{3}$ radians is subtended in a circle of radius 6 cm. Find the arc length.

10. Verify the identity: $(\sin(\theta) + \cos(\theta))^2 = 1 + 2\sin(\theta)\cos(\theta)$

11. Classify the triangle (ASA or SAA) and solve: A = 60°, B = 15°, c = 6. Find the missing angle and remaining sides.

12. Two fire towers are located 10 miles apart. Tower A observes a fire at angle 71° and Tower B at angle 56°. Find the distance from each tower to the fire.

13. Use a sum-to-product formula to express cos(5θ) − cos(3θ) as a product. Simplify completely.

14. Use a double-angle identity to find $\sin^2(22.5°)$

15. Find the exact value of cos $(\frac{\pi}{8})$

16. Find the exact value of each expression:
    a) $\sin^{-1}(-\frac{1}{2})$
    b) $\cos^{-1}(\cos(\frac{7\pi}{6}))$

17. For $y = 2\sec(x - \frac{\pi}{4})$
    a) state the period
    b) identify the phase shift
    c) write the equations of two consecutive asymptotes on $(-\pi, \pi)$
    d) describe how its graph relates to $y = 2\cos(x - \frac{\pi}{4})$, including where the maxima and minima of cosine correspond to features of the secant graph. Graph both functions.

18. Find the exact value of each expression:
    a) sin $(\frac{3\pi}{2})$

b) $\cos\left(-\frac{\pi}{6}\right)$

c) $\tan\left(\frac{5\pi}{4}\right)$

19. In a right triangle with legs 5 and 5, find all six trigonometric function values of the 45° angle. Rationalize denominators.

a) sin (45º)
b) cos (45º)
c) tan (45º)
d) csc (45º)
e) sec (45º)
f) cot (45º)

20. Determine the number of triangles and solve b = 4, c = 2, B = 29°. Solve all possible triangle(s).

21. A pilot flies 200 miles due east, then turns and flies 150 miles on a bearing of N 30° E. How far is the pilot from the starting point?

22. Find three additional polar representations of the point $(2, \frac{\pi}{3})$:

(a) $r > 0$ with θ outside $[0, 2\pi)$,
(b) $r < 0$ with $0 \le \theta < 2\pi$,
(c) $r < 0$ with $\theta < 0$.
Then convert $(2, \frac{\pi}{3})$ to rectangular coordinates.

23. Convert the rectangular point $(-\sqrt{3}, 1)$ to polar coordinates, with $r > 0$ and $0 \le \theta < 2\pi$.

24. Solve: $\tan^2\theta - 3 = 0$ on $[0, \pi)$. List all solutions

25. A sinusoidal function has amplitude 4, period 6π, phase shift $\frac{\pi}{2}$ to the right, and no vertical shift. Write its equation in the form $y = A\sin(\omega x - \varphi)$.

26. A wheel of radius 2 feet rotates at 4 revolutions per second. Find the angular speed in rad/sec, and the linear speed of a point on the rim in $\frac{ft}{sec}$. Express answers in terms of π.

27. If $\sec\theta < 0$ and $\tan\theta > 0$, identify the quadrant where θ lies. Then state the sign (+ or −) of ALL six trigonometric functions in that quadrant.

28. If $\sin \alpha = \frac{3}{5}$ (α in Quadrant I) and $\cos \beta = -\frac{5}{13}$ (β in Quadrant II), find $\sin(\alpha + \beta)$.

29. An engineer needs to find the shortest distance to cross a river. Given B = 30°, a = 300 ft, C = 100°, find the shortest crossing distance. Round to the nearest foot.

30. Solve the triangle: b = 5, c = 3, A = 100°. Find side *a* and the remaining angles.

31. Transform $r = 4 \sin \theta$ into rectangular form. Identify the type of curve (circle, line, etc.) and its center and radius.

32. Determine whether the statement below is true or false.
Force is an example of a vector

33. Two forces of magnitude 30 N and 60 N act on an object at angles of 30º and -45º with the positive x-axis. Find the direction and magnitude of the resultant force, that is, find $\mathbf{F_1 + F_2}$

34. Two angles of a triangle are A = 50° and C = 80°. If the side between them is b = 12, find sides a and c.

35. For $y = -4\cos(3x + \frac{\pi}{2})$, what is the amplitude, period, phase shift — state direction and magnitude, maximum and minimum values, the x-value of the first maximum for $x \geq 0$.

36. Verify the identity: $\frac{\sec^2\theta - 1}{\sec^2\theta} = \sin^2\theta$

37. Find the exact value of $\sin(\frac{5\pi}{12})$ using the sum formula

38. Simplify the expression: $\sin\theta \cdot \csc\theta - \sin^2\theta$

39. Find the direction angle of **v** for the following vector. v= 9i + 9j
What is the direction angle of v?

40. Transform r = 4 sin θ into rectangular form. Identify the type of curve, its center, and its radius.

41. The displacement *d* (in meters) of an object at time *t* (in seconds) is given by the following equation: $d = 5\cos(4t)$
    a) Is the motion simple harmonic?
        - No, because the given equation is not in the form of $d = A\cos(\omega t)$
        - No because the given equation involves a cosine function instead of a sine function
        - Yes, because the given equation is in the form of $d = A\cos(\omega t)$
        - Yes, because the given equation is in the form $d = ae^{-\frac{bt}{2m}}\cos(\sqrt{\omega^2 - \frac{b^2}{4m^2}}t$
    b) What is the maximum displacement from its resting position?

    c) What is the time required for one oscillation?

    d) What is the frequency?