Problem Set 1

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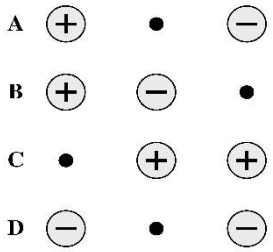
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Charges and Electric Forces

1. A certain lightning bolt moves 43.5 C of charge. How many fundamental units of charge is this?
   Ans: 2.715E20 units

2. What is the net charge (in coulombs) of the following ionized Carbon atom $^{12}_{6}C^{-2}$? Hint: here's a webpage describing how to read nuclear symbols: link
   Ans: -3.204E-19 C

3. In four different cases, shown below, a small positive charge is placed near two fixed charges. Rank the cases from least to greatest for the force experienced by the small charge.



   Ans: D, B, C, A

4. In a salt crystal, the distance between adjacent sodium and chloride ions is 2.82 Angstroms (or .282 nm; refer to the supplemental practice document for refreshers on these units). What is the force of attraction between the two singly charged ions?
   Ans: 2.901E-9 N

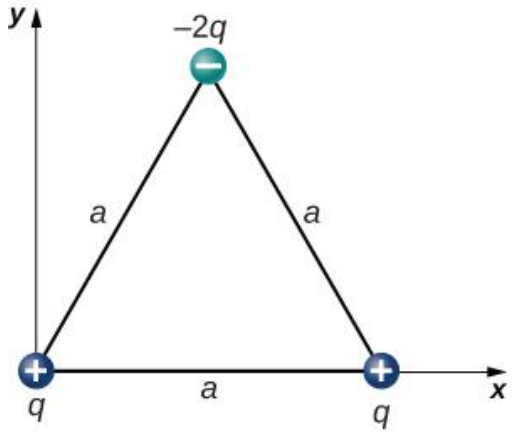
5. I have a charge with 3 μC and a charge with -5.2 μC a certain distance apart. What is this distance if the force between them is 1.00 N?
   Ans: 0.375 m

6. At what distance apart do two electrons need to be to each experience an electric force equal to the force each experiences due to gravity from the Earth?
   Ans: 5.087 m

7. Three point charges are placed on the x-axis, 2 μC at x = 0 m, -3 μC at 40 cm, and -5 μC at x = 120 cm. Find the magnitude and direction of the force on the -3 μC charge.
   Ans: 0.55 N to the left

8. There are two charges q_1 = 35 μC and q_2 = 50 μC placed 0.5 m apart and fixed in place. Where should the charge q_3 = 15 μC be placed so that it experiences no force on it?
Ans: .228 m from q_1 and .272 from q_2

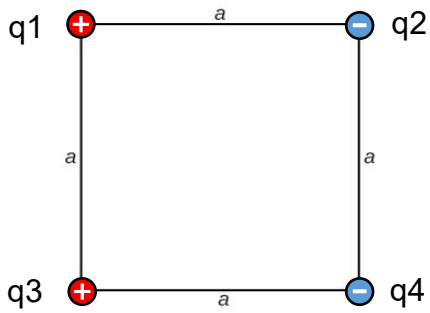
9. What is the net electric force on the charge located at the top of the triangle show below?



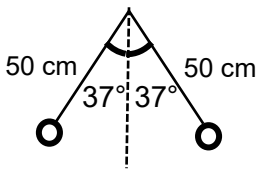
Ans: $\frac{2\sqrt{3}q^2k}{a^2}$ in the negative y direction

10. Four particles form a square with side lengths 'a' and have charges q1 = +q, q2 = −2q, q3 = +2q, and q4 = −q. a) What is the net electric force on q1 and in what direction? b) What if q = 3 μC and a = 3 mm?

Ans: $F_{net} = \frac{kq^2}{a^2}\left(\left(2+\frac{1}{2\sqrt{2}}\right)\hat{\imath} + \left(\left(2-\frac{1}{2\sqrt{2}}\right)\hat{\jmath}\right.\right.$, θ = 35°, b) 2.58E4 N, θ = 35°



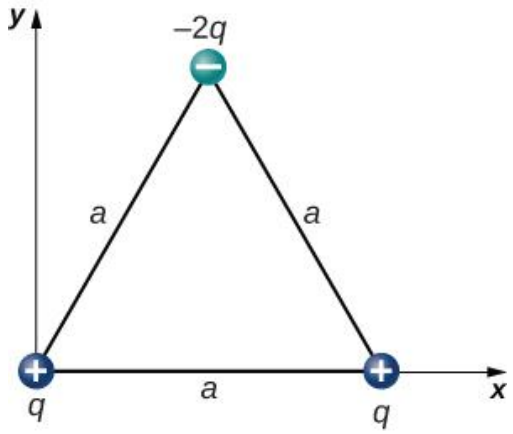
11. The two balls shown in the figure below have identical masses of 0.20 g each. When suspended from 50 cm long strings, they make an angle of 37° to the vertical. If the charges on each are the same, how large is each charge?
Ans: 0.24 μC



Electric Fields

1. What is the value of the electric field 5 m away from a single electron?
   Ans: 5.767E-11 N/C

2. What is the net electric field on the charge located at the top of the triangle show below?



   Ans: $\frac{\sqrt{3}qk}{a^2}$ in the positive y direction

3. If q_1 is located at x = 3m, can you arrange the two point charges $q_1 = 6.7\times10^{-6}$C and $q_2 = 4.0\times10^{-6}$C along the x-axis so that E=0 at the origin?
   Ans: Charge 2 located at -2.318 m

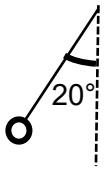
4. If q_1 is located at x = 3m, can you arrange the two point charges $q_1 = -6.7\times10^{-6}$C and $q_2 = 4.0\times10^{-6}$C along the x-axis so that E=0 at the origin?
   Ans: Charge 2 located at +2.318 m

5. (a) What is the electric field of an oxygen nucleus at a point that is $5.6\text{x}10^{-10}$m from the nucleus? (b) What is the force this electric field exerts on a second oxygen nucleus placed at that point? Hint: How many protons are in the nucleus?
   Ans: a) 3.678E10 N/C
   b) 4.714E-8 N

6. A proton is suspended in the air by an electric field where the acceleration due to gravity is g. What is the strength of this electric field?
   Ans: 1.02E-7 N/C

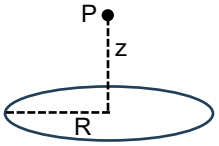
7. On a typical clear day, the atmospheric electric field points downward and has a magnitude of approximately 100 N/C. Compare the gravitational and electric forces on a small dust particle of mass 3.5×10^{-19} kg that carries a single electron charge. What is the acceleration (both magnitude and direction) of the dust particle?
   Ans: 35.97 m/s^2 in the positive y direction

8. A tiny ball with a mass of 0.6 g and a charge of -3.1 μC is held at the end of a piece of string as shown below with an angle of 20° to the vertical. If the electric force on the ball points directly to the left, what is the strength of the electric field and what direction does it point?
   Ans: 700 N/C, positive x-direction



9. An electron is launched with an initial velocity of $3x10^6$ m/s. It travels 45 cm before stopping due an electric field in the region. Find the direction and magnitude of the electric field.
   Ans: 57 N/C in the positive x-direction

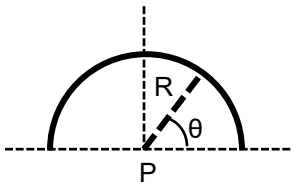
10. Derive the equation for the electric field a distance z above the center axis of a ring of charge with a uniform charge density of λ.
    Ans: $\vec{E} = \frac{kQz}{(z^2+R^2)^{\frac{3}{2}}}\hat{j}$



11. Derive the equation for the electric field a distance z above an infinite line of charge that carries a uniform charge density of λ.
    Ans: $\vec{E} = \frac{2k\lambda}{z}\hat{j}$

12. A line of charge has a charge density given by λ = ln(y). The line of charge starts at the origin and ends at y = 10 m. What are the integrals for the components of the electric field a distance x to the right of the origin?
    Ans: $\overrightarrow{E_x} = \int_0^{10} \frac{kxln(y)dy}{(x^2+y^2)^{\frac{3}{2}}}$, $\overrightarrow{E_y} = \int_0^{10} \frac{kyln(y)dy}{(x^2+y^2)^{\frac{3}{2}}}$

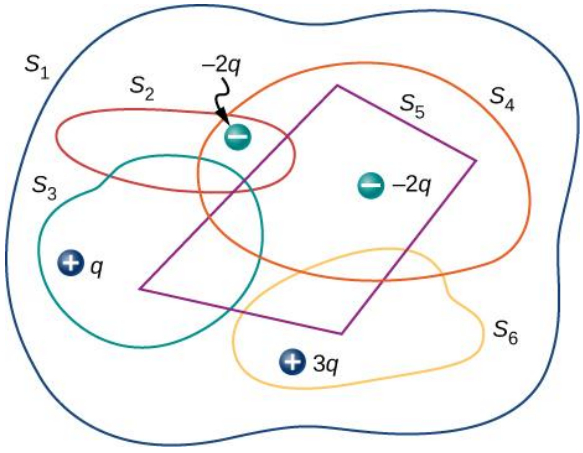
13. A line of positive charge is shaped into a semicircle with radius R (see figure). The non-uniform electric charge per unit length along the semicircle is given by $\lambda = \lambda_0 \sin\theta$.
    a. What is the total charge Q of the semicircle? b. What's the electric field at point P?
    Ans: a) $2\lambda_0 R$, b) $\vec{E} = -\frac{\pi k\lambda_0}{2R}\hat{j}$



Gauss' Law and Electric Potential Energy

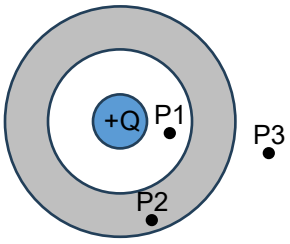
1. What is the electric flux through a 2.0 x 2.0 plane that has an area vector parallel to a uniform electric field of magnitude 5.6×10^6 N/C? b) What if the plane's area vector was perpendicular to E instead?
   Ans: a) 2.24E7 N·m²/C, b) 0 N·m²/C

2. Determine the electric flux through each surface whose cross-section is shown below.



   Ans: S_1: 0
   S_2: $-2q/\varepsilon_0$ Nm²/C
   S_3: q/ε_0 Nm²/C
   S_4: $-4q/\varepsilon_0$ Nm²/C
   S_5: $-2q/\varepsilon_0$ Nm²/C
   S_6: $3q/\varepsilon_0$ Nm²/C

3. A small metal sphere carrying charge +Q is located at the center of a spherical cavity in a large uncharged metal sphere. Find the electric field a) inside the small metal sphere, b) at P1, c) at P2, d) and at P3 shown below in the image to the right.
   Ans: a) 0 N/C, b) $E = kQ/r^2$, c) 0 N/C, d) $E = kQ/r^2$

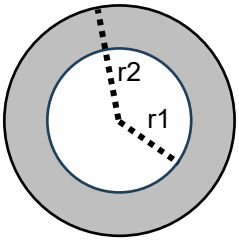


4. What is the value of the electric field .5 m from a square piece of metal which is 1 km x 1 km and has a net charge of 1 C?
   Ans: $E = 5E\text{-}7/\varepsilon_0$ N/C

5. A long silver rod of radius 3 cm has a charge of −5μC/cm on its surface. (a) Find the electric field at a point 5 cm from the center of the rod (an outside point). (b) Find the electric field at a point 2 cm from the center of the rod (an inside point).
   Ans: a) 1.8E8 N/C radially inward, b) 0 N/C

6. A sphere of radius 5 m has charge of 3.6 nC which is distributed uniformly. What is the electric field at the following distances from the center of the sphere? a) 1.2 m, b) 5 m, c) 7 m
   Ans: a) 0.31 N/C, b) 1.30 N/C, c)0.66 N/C

7. A total charge Q is distributed uniformly throughout a spherical shell of inner and outer radii r_1 and r_2, respectively. Find the electric field a distance 'r' when r is a) inside the inner radii ($r < r_1$), b) between the inner/outer radii ($r_1 < r < r_2$), and c) outside the outer radii ($r_2 < r$),.
   Ans: a) E = 0, b) $\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\frac{r^3 - r_1^2}{r_2^3 - r_1^3}\hat{r}$ **, c)** $\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$



8. Charge is distributed throughout a spherical volume of radius R with a density $\rho=\alpha r^2$, where α is a constant. Determine the electric field due to the charge at points both a) inside and b) outside the sphere.
   Ans: a) $\vec{E} = \frac{\alpha r^3}{5\varepsilon_0}$**, b)** $\vec{E} = \frac{\alpha R^5}{5\varepsilon_0 r^2}$

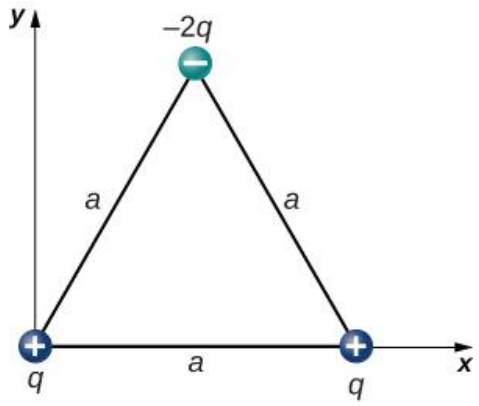
9. Two point charges, with respective charges q_1 and q_2, are initially positioned on the x-axis at positions 0 and x_i, respectively. How much work must be done to move the charge q_2 from x_i to a new position x_f?
   Ans: $\frac{kq_1q_2}{x_f} - \frac{kq_1q_2}{x_i}$

10. An electron is released from rest a distance z above a positively charged infinite plane (charge density σ). What is the speed of the electron just before it hits the surface? Use q_e for the charge of an electron
    Ans: $\sqrt{\frac{\sigma q_e z}{m\varepsilon_0}}$

11. To form a hydrogen atom, a proton is fixed at a point and an electron is brought from far away to a distance of 0.529×10^{-10}m, the average distance between proton and electron in a hydrogen atom. How much work is done?
    Ans: 4.37E-18 J

12. What is the net electric potential energy of the following 3 charge system?

Ans: $-3\frac{kq^2}{a}$



13. Two charges each with charge q are a distance 'r' apart from each other. a) What is the energy in this two charge system? b) What is the energy of the system if a third charge, 2q, is brought in to form an equilateral triangle with the other two charges.
    Ans: a) $\frac{kq^2}{r}$**, b)** $5\frac{kq^2}{r}$

14. An evacuated tube uses an accelerating voltage of 44 kV to accelerate electrons to hit a copper plate and produce X-rays. Ignoring relativistic effects, what would be the maximum speed of these electrons?
    Ans: 1.244E8 m/s

15. I want to accelerate an electron to the speed of sound (343 m/s) by electrostatic force alone. To do so I've devised a way to store as much charge as I'd like at a single fixed point which will be located 0.35 m from the electron. How much charge do I need to accumulate to accomplish my goal?
    Ans: 1.301E-17 C

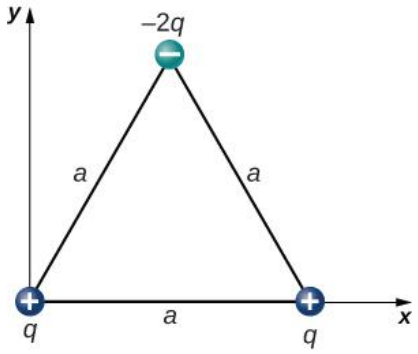
Electric Potential and Capacitors

1. How far from a 3.50-μC point charge is the potential 75 V? At what distance is it 3.00×10^3 V?
   Ans: a) r = 420 m, b) r = 10.5 m

2. If the potential due to a point charge is 8.00×10^3V at a distance of 12.0 m, what are the sign and magnitude of the charge?
   Ans: q = 1.067E-5 C

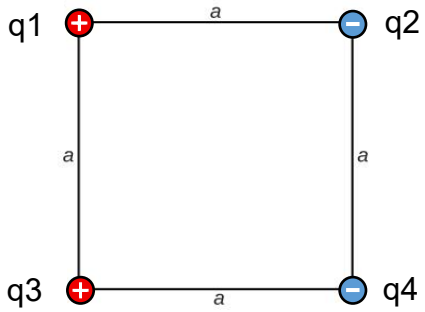
3. There are two charges q_1 = 35 μC and q_2 = -50 μC fixed in place on the x-axis. q_1 is located at x=0 and q_2 is located at x=0.5. What is the location on the x-axis where the potential equals 0?
   Ans: Distance from charge 1 = .206 m, Distance from charge 2 = .294 m

4. What is the net electric potential at the top of the triangle show below?



   Ans: V = 2kq/a

5. Four particles form a square with side lengths ‘a’ and have charges q1 = +1 C, q2 = −2q, q3 = +2q, and q4 = −q. a) What is the net electric potential where q1 is located? b) What if q = 3 μC and a = 3 mm?
   Ans: a) $V = -\frac{kq}{\sqrt{2}a}$**, b) V = -6.36E6 V**



6. Derive the equation for the electric potential a distance z above an infinite line of charge that carries a uniform charge density of λ. Since the charge densities extend to infinity, the potential won’t be 0 at infinity, so instead we will set the potential equal 0 at R_0.
   Ans: $V = 2k\lambda ln(\frac{R_0}{z})$

7. Two parallel plates 10 cm on a side are given equal and opposite charges of magnitude 5.0×10^{-9} C. The plates are 1.5 mm apart. What is the potential difference between the plates?
   Ans: V = 84.706 V or 7.5e-10/ε_0 V

8. Find the electric potential inside and outside of a uniformly charged sphere with net charge of Q and a radius of R.
   Ans: r < R: $V = \frac{kQ}{2R}\left(3 - \frac{r^2}{R^2}\right)$**, r > R:** $= \frac{kQ}{R}$

9. Calculate the voltage applied to a 2.00-μF capacitor when it holds 3.1μC of charge.
   Ans: 1.55 V

10. A proton is placed at the positive plate inside a parallel plate capacitor with a potential difference of 100 V across them. How fast will the proton be going when it reaches the negative plate?
    Ans: 1.4E5 m/s

11. An anxious physicist worries that the two metal shelves of a wood frame bookcase might obtain a high voltage if charged by static electricity, perhaps produced by friction. (a) What is the capacitance of the empty shelves if they have area $1\times10^2 m^2$ and are 0.200 m apart? (b) What is the voltage between them if opposite charges of magnitude 2.00 nC are placed on them? (c) To show that this voltage poses a small hazard, calculate the energy stored.
    Ans: a) 4.43E-9F
    b) .453 V
    c) 4.53E-10 J

12. How much surface area is necessary to make a capacitor with a capacitance of 1 F assuming we have a gap of 10 μm and are able to maintain a vacuum in the space

between the plates? How much surface area is necessary if we add a dielectric with a dielectric constant of 5.0?

Ans: a) 1.129E6 m^2

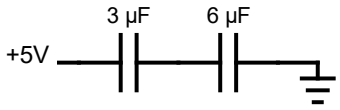
b) 2.26E5 m^2

13. A parallel-plate capacitor consists of two plates that have an area of 200 cm^2 each and are separated by a 0.4 cm air filled gap. If the capacitor is connected to a 500 V source, a) what is the charge on it, b) what is the energy stored in it, c) what is the electric field between the plates? If a liquid with k = 2.6 is poured between the plates, how much additional charge will go onto the plates?

    Ans: a) 44 pF, b) 22nC, c) 5.5 µJ, d) 125 kV/m, e) 35 nC

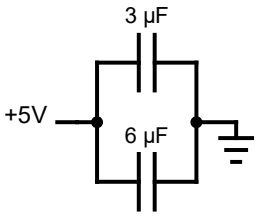
14. a) What is the equivalent capacitance in the following circuit? b) What is the charge stored on each capacitor? c) What is the voltage across each capacitor?

    Ans: 2 µF b) Q_3 = 10 µC, Q_6 = 10 µC, c) V_3 = 3.33 V, V_6 = 1.67 V



15. a) What is the equivalent capacitance in the following circuit? b) What is the charge stored on each capacitor? c) What is the voltage across each capacitor?

    Ans: 9 µF b) Q_3 = 15 µC, Q_6 = 30 µC, c) V_3 = 5 V, V_6 = 5 V

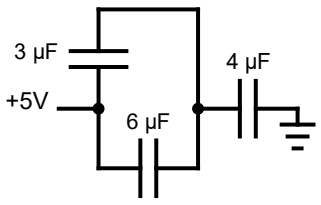


16. a) What is the equivalent capacitance in the following circuit? b) What is the charge stored on each capacitor? c) What is the voltage across each capacitor?

    Ans: a) 2.77 µF

    b) Q_3 = 4.62 µC, Q_6 = 9.23 µC, Q_4 = 13.85 µC,

    c) V_3 = 1.54 V, V_6 = 1.54 V, V_4 = 3.46 V



Current and Resistance

1) Assuming we maintained a constant current of 5 A, how long would it take to transfer 5 C of charge?
   Ans: 1 second

2) The current through a conductor is modeled as $I(t) = I_0 \cos(2\pi[60\ \text{Hz}]t)$. Write an equation for the charge as a function of time.
   Ans: $\frac{\mathbf{I_0}}{\mathbf{2\pi 60 Hz}}(\mathbf{\sin(2\pi[60\ Hz]t)})$

3) A cathode ray tube (CRT) is a device that produces a focused beam of electrons in a vacuum. The electrons strike a phosphor-coated glass screen at the end of the tube, which produces a bright spot of light. The position of the bright spot of light on the screen can be adjusted by deflecting the electrons with electrical fields, magnetic fields, or both. Although the CRT tube was once commonly found in televisions, computer displays, and oscilloscopes, newer appliances use a liquid crystal display (LCD) or plasma screen. You still may come across a CRT in your study of science. Consider a CRT with an electron beam average current of 25.00μA. How many electrons strike the screen every minute?
   Ans: 9.37E15 electrons

4) A 0.2 mm diameter copper wire is sealed end to end to a 5 mm diameter iron rod. If a current of 8 A is sent through the copper wire, a) what is the current and current density in the iron rod and b) what is the current density in the copper wire?
   Ans: a) 8 A, 407 kA/m^2, b) 2.55E8 A/m^2

5) (a) To what temperature must you raise a copper wire (ρ_{copper}= 1.68E-8 Ω·m), originally at 20.0°C, to double its resistance, neglecting any changes in dimensions? (b) Does this happen in household wiring under ordinary circumstances?
   Ans: a) T = 276.4 °C, b) It shouldn't, at 150 °C wood can catch fire

6) What is the resistance of a 20.0-m-long piece of 12-gauge copper wire having a 2.053-mm diameter? ρ_{copper}= 1.68E-8 Ω·m
Ans: 0.102 Ω

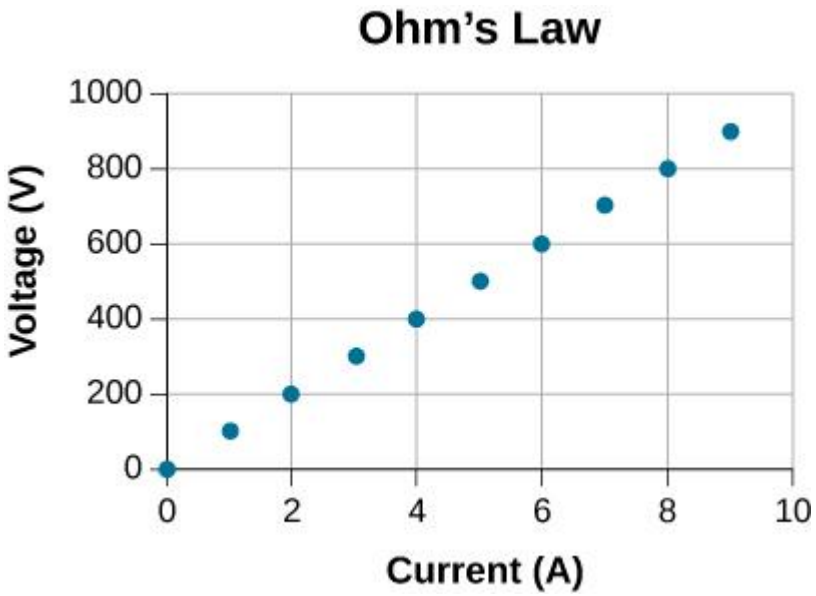
7) What current flows through the bulb of a 3.0-V flashlight when its hot resistance is 3.6Ω?
Ans: 830 mA

8) A resistor is connected in series with a power supply of 20.00 V. The current measure is 0.50 A. What is the resistance of the resistor?
Ans: 40 Ω

9) A current of 3.0 A flows down a straight metal rod that has a 0.2 cm diameter and a length of 1.5 m. If a potential difference of 40 V is placed between its lengths, what is the a) current density, b) electric field in the rod, c) resistivity of the material of the rod.
Ans: a) 9.55E5 A/m^2, b) 27 V/m, c) 28 μΩ·m

10) What current flows through a rod with diameter *d* that has a resistivity of ρ and a length of *L* when a potential difference *V* is applied across it?
Ans: $I = V\pi d^2/(4\rho L)$

11) A resistor is placed in a circuit with an adjustable voltage source. The voltage across and the current through the resistor and the measurements are shown below. Estimate the resistance of the resistor.



Ans: 100 Ω

12) What is the maximum voltage that can be applied to a 20-kΩ lightbulb rated at 0.25W?
Ans: 70.7 V

13) A heater is being designed that uses a coil of nichrome wire ($\rho = 100 \times 10^{-8}\ \Omega \cdot m$) to generate 300 W using a voltage of V=110V. The wire is 14-gauage and therefore has a diameter of 1.627 mm. How long should the engineer make the wire?
Ans: 83.85 m

14) An alternative to CFL bulbs and incandescent bulbs are light-emitting diode (LED) bulbs. A 100-W incandescent bulb can be replaced by a 16-W LED bulb. Both produce 1600 lumens of light. Assuming the cost of electricity is $0.10 per kilowatt-hour, how much does it cost to run each bulbs for one year if they each run for four hours a day?

Ans: LED: $2.34, Incandescent: $14.6