Electric Potential and Electric Field – Highlights

College Physics II (PHY 2054) with Ashiya

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Electric Potential Energy and Potential Difference

Recall from Physics I:

- Kinetic energy is characterized as the energy of motion, while potential energy is characterized as the energy of rest.
- Law of Conservation of Energy: the total energy in a system is conserved if there is no net addition or subtraction of work. Therefore, the total mechanical energy (kinetic energy + potential energy) remains constant.
    - As an object is moved, its potential energy gets converted to kinetic energy
    - The amount of potential energy and kinetic energy in a system have an inverse relationship
- The further an object is from where it wants to go (based on the forces acting on it), the more potential energy that object has.
- Energy has units of Joules (J)

As a charge is accelerated by an electric field, its electric potential energy is converted to kinetic energy

- Work done to move the charge results from a loss of potential energy, so:

$$W = -\Delta PE$$

- A positive charge would have high potential energy and low kinetic energy in a region close to positive charges. It would have low potential energy and high kinetic energy in a region close to negative charges
- A negative charge would have high potential energy and low kinetic energy in a region close to negative charges. It would have low potential energy and high kinetic energy in a region close to positive charges

We now consider another measure called Electric Potential, which is not the same as Electric Potential Energy, but instead is Electric Potential Energy per unit charge:

$$V = \frac{PE}{q}$$

Where: V = Electric Potential, in Joules/Coulomb (J/C) otherwise known as Volts (V)

PE = Potential Energy, in Joules (J)
q = Charge, in Coulombs (C)

Think of the relationship between Electric Potential Energy (PE) and Electric Potential as being analogous to the relationship between Electric Force and Electric Field, respectively.

Just as Electric Field is not force, but *reflects* electric force, in terms of a positive charge, Electric potential is not energy, but *reflects* electric potential energy of a positive charge.

This relationship can be seen in the units of each:

Electric Force = F	Electric Field = F/C
Electric Potential Energy = J	Electric Potential = J/C

As electric force is a vector, electric field is defined in terms of a positive charge using direction → Field lines point in the direction of force on a positive charge

Since potential energy is scalar (it has only magnitude, not direction), it is defined in terms of a positive charge using magnitude → Electric potential is high where electric potential energy of a positive charge would be high; Electric potential is low where electric potential energy of a positive charge would be low

- Because negative charges experience high electric potential energy in areas where positive charges experience low electric potential energy → negative charges have high PE in areas of low potential, and low PE in areas of high potential

Because it can be complicated to determine the actual electric potential of a region, we instead generally think about potential relationally or by comparing two separate regions to each other and looking at the difference in potential between the regions.

Instead of trying to calculate that region A has an electric potential of 110 V, or that region B has an electric potential of 120 V, we singularly calculate and work with the fact that there is a difference of 10 V between them.

Potential Difference, also known as Voltage, is the difference in Electric Potential between two regions:

$$\Delta V = V_B - V_A$$

Where: ΔV = Potential Difference, in Volts
V_B = Electric potential of the region the charges go to, in Volts
V_A = Electric potential of the region charges leave from, in Volts
If you have to determine for yourself which region is V_A and which is V_B, you must consider whether the charges you are dealing with are negative or positive

For the relationship between potential difference and electric potential energy:

$$\Delta V = \frac{\Delta PE}{q}$$

Where: ΔV = Electric potential difference, in Volts
ΔPE = Change in Electric Potential Energy (energy 'delivered' or lost), in Joules
q = Charge (can be negative or positive), in Coulombs

An Electron Volt (eV) is an energy unit used to define the energy given to a fundamental charge (usually a single proton or electron) by accelerating it through a potential difference of 1 V:

$$(1.60 \times 10^{-19}\ C)(1\ V) = 1.60 \times 10^{-19}\ \text{J}$$

$$1\ eV = 1.60 \times 10^{-19}\ Joules$$

- Any single electron given *x* eV of energy, would have been accelerated through a potential difference of *x* V ….

  Ex. Accelerating an electron through a potential difference of 20 V, gives the electron 20 eV of energy

Electric Potential and Electric Field

The Relationship between Voltage and Electric Field for a Uniform Electric Field ONLY (which is found between two parallel metal plates):

$$V_{AB} = Ed$$

Where: $V_{AB} = V_A - V_B$
V_A = Electric potential of the region positive charges leave from (higher V), in Volts
V_B = Electric potential of the region positive charges go to (lower V), in Volts
d = distance between the two plates, in meters
E = magnitude of the Electric Field, in Newtons/Coulomb

- Rearrangement of this equation shows that the units of Electric Field can also be given in Volts/meter (V/m)

In general (uniform field or not), electric field is said to be a gradient of electric potential, as electric fields point in the direction of decreasing potential

- Remember: E field lines point away from positive charges and towards negative charges; Regions of positive charge are regions of high potential, and regions of negative charge are regions of low potential

- This relationship between Electric Field and Potential takes equation form as:

$$E = -\frac{\Delta V}{\Delta s}$$

Where: E = magnitude of the Electric field in Volts/meters or Newtons/Coulomb
ΔV = the change in potential across distance s, in Volts
Δs = distance over which the difference in potential takes place, in meters

Electric Potential and Point Charges

We said previously that it can be complicated to exactly calculate electric potential, so we tend to just find potential difference by relating one region to another. However, we can calculate the Electric potential generated by a point charge at a specific distance away from that point charge by assuming that the potential is zero at a distance of ∞ away from the point charge.

- Electric Potential of a point charge:

$$V = \frac{kQ}{r}$$

Where: V = Potential at distance r, in Volts
k = $8.99 \times 10^9 \frac{N \cdot m^2}{C^2}$ or 9.0×10^9 ; USE WHICHEVER YOUR PROFESSOR PREFERS
Q = charge of the point charge, in Coulombs
r = distance of the point of interest from the point charge, in meters

Notice that while this equation looks very similar to those for Electric Force and Electric Field, there is one very important difference: No absolute value signs! This means that:

- You have to plug charge into the equation with its corresponding sign
- Negative charge generates a negative potential and positive charge generates a positive potential
    - This should make sense: We said previously that like charges repel each other. We also said earlier that negative charges move away from areas of negative potential, and that positive charges move away from areas of high potential → Point charges generate potentials that attract opposite charges and drive away like charges

On calculating Total Potential:

Remember that Electric Potential is not a vector, and thus has no direction. This means that unlike calculating net Electric Force and net Electric field, we won't have to consider direction when trying to find the total Electric Potential at a given point. You can just add them outright. However, make sure to include the positive and negative signs of the values given by the equation.

Equipotential Lines

Notice from the equation for Electric Potential of a point charge and the equation for the relationship between Electric Field and Potential that potential at a certain point is dependent upon distance from the generating thing (whether charged plate or point charge). → Potential should be the same at all points of the same distance from the generating thing. The line these points generate is known as an equipotential line.

- At all points along a given equipotential line, potential is the same value (or "equipotential") because all of those points are the same distance from the generating thing
    - When we're dealing with a point charge, these points form a circular line around the point charge → equipotential lines are increasingly large circles, with the point charge at the absolute center
    - When we're dealing with charged plates, these points form a line that runs parallel to the plates → equipotential lines run parallel to the plates throughout the distance between the plates
- Equipotential lines run perpendicular to electric field lines
- Since the potential is the same along an equipotential line, the potential difference between any two points on the line is 0 → no work is required to move a charge along an equipotential line

Capacitors

- Made up of parallel conducting plates into which equal amounts of charge are separated (one is charged with -Q Coulombs, and the other is charged with +Q Coulombs)
    - → remain overall neutral, but referred to as storing Q charge
- Are initially uncharged until the plates are connected to the two terminals of a battery
- Once charged, a uniform Electric Field arises between the two charged plates; the greater the charge, the greater the magnitude of the field
- The amount of charge a capacitor can store is dependent on the voltage applied to the capacitor via the battery it is hooked up to, and the capacitor's physical characteristics
    - A capacitor's ability to hold charge based on its physical characteristics is measured in charge stored per volt, known as Capacitance, the units of which are Coulombs/Volt (C/V) otherwise known as Farads (F)
    - The relationship between voltage applied, capacitance, and charge is:

$$Q = CV$$

Where: Q = Charge stored, in Coulombs (C)
C = Capacitance, measured in Coulombs/Volt (C/V) or Farads (F)
V = Voltage applied to capacitor, measured in Volts (V)

- Capacitance of a given capacitor can be calculated according to the following equation:

$$C = \varepsilon_0 \frac{A}{d}$$

Where: C = Capacitance, in Farads (F) or Coulombs/Volt (C/V)
ε_0 = Permittivity of Free Space constant, $8.85 \times 10^{-12} \frac{F}{m}$
A = area of once capacitor plate, in meters squared (m^2)
d = distance between the two plates, in meters (m)

Dielectrics

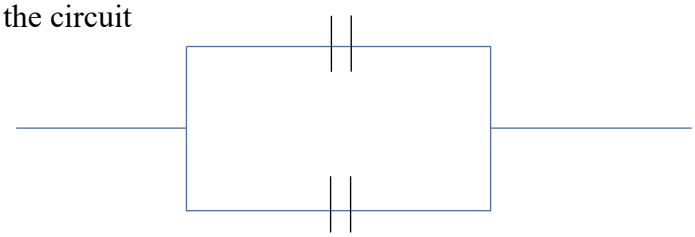
- Dielectrics are insulators that are placed in between the parallel plates of a capacitor to increase Capacitance
  - We see from the capacitance equation above that a decrease in distance between the plates increases the capacitance of a capacitor. However, as we bring capacitor plates closer and closer together, we cross a certain threshold at which charges begin jumping between the plates and we no longer have separation of charge → no longer have a capacitor
  - As insulators, dielectrics allow for capacitor plates to be brought closer together while maintaining the separation of charge by preventing this 'breakdown' from happening
    - → smaller distance → higher capacitance
- When calculating Capacitance of a capacitor with a dielectric we use a slightly modified equation:

$$C = \kappa \varepsilon_0 \frac{A}{d}$$

Where: C = Capacitance, in Farads (F) or Coulombs/Volt (C/V)
ε_0 = Permittivity of Free Space constant, $8.85 \times 10^{-12} \frac{F}{m}$
A = area of once capacitor plate, in meters squared (m^2)
d = distance between the two plates, in meters (m)
κ = dielectric constant, unitless, dependent on which material dielectric is made of

Capacitors in Circuits

- Multiple capacitors can be hooked up to a single battery in combinations of two types of connections:
  - Parallel: Capacitors are in parallel when they are on wires that run alongside each other, or rather when a charge that runs through one will not run through the other to get to the 'end' of the circuit



- Series: Capacitors are in series when they come one right after another on the circuit, or rather, when a charge has no choice but to go through one if it has gone through the other



!!!This can be confusing!!! Please refer to the session recording on the WordPress that deals with this topic as this is something needs to be understood visually

- Total Capacitance, or Equivalent Capacitance, is a calculated value that represents the capacitance of a single capacitor that replaced and had the same effect of all of the capacitors in either a given parallel formation or a given series formation.
  - Total Capacitance for Capacitors in Series:

$$\frac{1}{C_E} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \ldots$$

Where: C_E = Equivalent Capacitance, in Farads
C_n = Individual Capacitance of each Capacitor, in Farads

  - Total Capacitance for Capacitors in Parallel:

$$C_E = C_1 + C_2 + C_3 \ldots$$

Where: C_E = Equivalent Capacitance, in Farads
C_n = Individual Capacitance of each Capacitor, in Farads

  - When you have combinations of parallels and series formations, you have to work your way outwards. Calculate the equivalent capacitance of your innermost groups first, and then use those calculated values for your next 'level' of calculations.
    !!!This can be confusing!!! Please refer to the session recording on the WordPress that deals with this topic as this is something needs to be understood visually

Energy Stored in Capacitors

There are three different equivalent equations you can use that relate energy stored in a capacitor to voltage, charge or capacitance. Choose which one to use in a given situation based on what values you have or need.

Energy Stored in a Capacitor:

$$E_{cap} = \frac{QV}{2}$$

$$E_{cap} = \frac{CV^2}{2}$$

$$E_{cap} = \frac{Q^2}{2C}$$

Where: E_{cap} = Energy stored in/delivered by the capacitor, in Joules (J)

C = Capacitance, in Farads (F)

V = Voltage of capacitor, in Volts (V)

Q = Charge of the capacitor, in Coulombs (C)

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