# Surrogate to Poincaré inequalities on manifolds for collective dimension reduction in nonlinear feature spaces

Alexandre Pasco
LMJL, École Centrale de Nantes, Nantes Université
alexandre.pasco@ec-nantes.fr

Our goal is to approximate a family $(u(\cdot, y))_{y\in\mathcal{Y}}$ of continuously differentiable functions $u(\cdot, y) : \mathbb{R}^d \to \mathbb{R}$, parametrized by some set $\mathcal{Y}$ equipped with some probability distribution, using a collective dimension reduction approach. The approximation format we consider is a composition of functions,

$$\hat{u}(\cdot, y) := f(g(\mathbf{x}), y),$$

where $g : \mathbb{R}^d \to \mathbb{R}^m$, $m \leq d$, and $f : \mathbb{R}^m \times \mathcal{Y} \to \mathbb{R}$ are built in a two stage procedure. For a fixed $g$, we build $f$ using classical regression methods, involving evaluations of $u$. Recent works [1, 2] proposed to build a nonlinear $g$ by minimizing a loss function derived from Poincaré inequalities on manifolds, involving evaluations of the gradient of $u$. A problem is that minimizing this loss function may be a challenging task. Hence in this work, we introduce new quadratic surrogates to this objective function. Leveraging concentration and anti-concentration inequalities, we provide quasi-optimality results on our surrogates, for a class of functions $g$, including polynomials, and a wide class of input probability measures.

We investigate performances on different benchmarks for various training sample sizes. We show that using our surrogates as a warm start of a standard iterative method for minimizing the training Poincaré inequality based loss yields significant improvement, especially for small training sets.

**Joint work with:** Anthony Nouy (LMJL, École Centrale de Nantes, Nantes Université).

## References

[1] D. Bigoni, Y. Marzouk, C. Prieur, and O. Zahm. Nonlinear dimension reduction for surrogate modeling using gradient information. *Information and Inference: A Journal of the IMA*, 11(4):1597–1639, 2022.

[2] R. Verdière, C. Prieur, and O. Zahm. Diffeomorphism-based feature learning using Poincaré inequalities on augmented input space. *Journal of Machine Learning Research*, 26(139):1–31, 2025.